1. Recall the definitions of the "floor" and "ceiling" functions ($\lfloor\ \rfloor$ and $\lceil\ \rceil$, respectively), from WS01Mon: for each $x \in \mathbb{R}$,

$$\lfloor x \rfloor = \max\{z \in \mathbb{Z} : z \leq x\},$$
$$\lceil x \rceil = \min\{z \in \mathbb{Z} : x \leq z\}.$$

(a) Let $x \in \mathbb{R}$. Express the definition of $\lfloor x \rfloor$ symbolically. In other words, write a symbolic expression that means: "$\lfloor x \rfloor$ is the maximum element in $\{z \in \mathbb{Z} : z \leq x\}$".

NOTE. Statements of the form "$e \in \{z \in \mathbb{Z} : z \leq x\}$" (where e is an expression) can be written more simply and directly as: "$e \in \mathbb{Z} \wedge e \leq x$".

Recall from WS05Wed that maximums are unique (if a set contains a maximum element, then it contains exactly one maximum element). So your answer is a *characterization* of $\lfloor x \rfloor$.

SOLUTION

$$\lfloor x \rfloor \in \mathbb{Z} \ \wedge\ \lfloor x \rfloor \leq x \ \wedge$$
$$\forall y \in \mathbb{Z},\ y \leq x \Rightarrow y \leq \lfloor x \rfloor$$

NOTE. The first line expresses the fact that $\lfloor x \rfloor \in \{z \in \mathbb{Z} : z \leq x\}$; the second line expresses the fact that $\lfloor x \rfloor$ is the maximum element in this set.