1. Prove each of the following statements. (NOTE: There is some repetition in what you are asked to prove, but each proof allows you to explore, and practice, different proof forms.)

(a) $\forall a, b \in \mathbb{N},\ a \geq 1 \wedge b \geq 1 \wedge (a \geq 2 \vee b \geq 2) \Rightarrow ab \geq 2$

SOLUTION

Let $a, b \in \mathbb{N}$.

Assume $a \geq 1$ and $b \geq 1$, and assume $a \geq 2$ or $b \geq 2$.

Case $a \geq 2$: Then $a \cdot b \geq 2 \cdot 1 = 2$.

Case $b \geq 2$: Then $a \cdot b \geq 1 \cdot 2 = 2$.

NOTE: this is the form of proof-by-cases represented by the following being equivalent:

$$\forall v \in D,\ P(v) \vee Q(v) \Rightarrow R(v)$$
$$(\forall v \in D,\ P(v) \Rightarrow R(v)) \ \wedge \ (\forall v \in D,\ Q(v) \Rightarrow R(v))$$

i.e., each of P and Q is sufficient for R iff their disjunction is.