1. For this problem, let $a, b \in \mathbb{N}$ and $x, y \in \mathbb{Z}$. Consider the following claims.

$$c_1(a,b) : a + b \neq 1 \qquad\qquad c_2(x,y) : x \cdot y < 0$$

Define the set $C_1 = \{(a,b) \in \mathbb{N}^2 : c_1(a,b)\}$, and similarly for C_2.

For *each* claim c_1, c_2, answer *each* of the following questions.

SOLUTION

SOME PROBLEM-SOLVING. *This was not required for a correct and complete solution,* but you are likely to find the problem much more difficult if you attempt it without first doing (at least some of) this.

test cases for $a + b \neq 1$:

$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$	$(2,0)$	$(0,2)$	$(2,1)$	$\cdots$
T	F	F	T	T	T	T	$\cdots$

So we see that for large enough values it's always true: any of $a \geq 2$, $b \geq 2$, or $(a \geq 1 \wedge b \geq 1)$ (and lots more that are less inclusive or more complicated than those) are sufficient. The third condition is the most inclusive and includes strictly more than the other two. To complete it we just add $(a = 0 \wedge b = 0)$.

To exclude $(0,1)$ we need to make sure that not both $a = 0$ and $b = 1$, i.e., at least one of $a = 0$ and $b = 1$ is false. That last "and" is *not* conjunction, it's just listing the conditions, saying at least one of the conditions in the set $\{a = 0, b = 1\}$ is false, which is an existential / disjunctive