Problem 1 (6 pts)**.** Find continuous solution

$$\begin{aligned} &u_{tt} - 16u_{xx} = 0, && , \ x > 0, -\infty < t < \infty \\ &u|_{t=0} = 0, \\ &u_t|_{t=0} = 16\cos(x), \\ &u_x|_{x=0} = 8\cos(4t). \end{aligned}$$

Solution. $u(x,t) = \phi(x+4t) + \psi(x-4t)$. Initial conditions imply $\phi(x) + \psi(x) = 0$, $\phi'(x) - \psi'(x) = 4\cos(x)$ as $x > 0$ and then $\phi(x) - \psi(x) = 4\sin(x)$, $\phi(x) = -\psi(x) = 2\sin(x)$ and

$$u(x,t) = 2\sin(x+4t) - 2\sin(x-4t) = 4\sin(4t)\cos(x) \qquad x > 4|t|.$$

Then, as $x = 0, t > 0$ boundary condition implies $2\cos(4t) + \psi'(-4t) = 8\cos(4t)$ and then $\psi(x) = 6\sin(x)$ as $x < 0$ and

$$u(x,t) = 2\sin(x+4t) + 6\sin(x-4t) \qquad 0 < x < 4t.$$