APM 346-2026S Solutions

Problem 1 (6 pts)**.** Find a solution of 3D-dimensional wave equation

$$u_{tt} - \Delta u = 0, \qquad (x, y, z) \in \mathbb{R}^3,\ -\infty < t < \infty, \tag{1.1}$$

$$u|_{t=0} = \sqrt{x^2 + y^2 + z^2}, \tag{1.2}$$

$$u_t|_{t=0} = 0. \tag{1.3}$$

Hint. Use spherical coordinates and consider $v = ru$.

Solution. Since problem (1.1)–(1.3) has a spherical symmetry and solution is unique, solution also has a spherical symmetry: $u = u(r, t)$ and our problem becomes

$$u_{tt} - \left(u_{rr} + \frac{2}{r}u_r\right) = 0,$$