Test 2. Solutions: Main Sitting

Problem 1 (6 pts)**.** Solve by Fourier method

$$
\begin{aligned}
&u_{tt} - u_{xx} = 0, && 0 < x < \pi,\\
&u|_{x=0} = u|_{x=\pi} = 0,\\
&u|_{t=0} = 0,\\
&u_t|_{t=0} = \begin{cases} 1 & 0 < x < \dfrac{\pi}{2},\\ 0 & \dfrac{\pi}{2} < x < \pi. \end{cases}
\end{aligned}
$$

Solution should be represented in the form of the appropriate Fourier series.

Solution. Plugging $u(x,t) = X(x)T(t)$ into equation and boundary conditions and separating variables we get

$$
\begin{aligned}
&X'' + \lambda X = 0,\\
&X(0) = X(\pi) = 0,\\
&T'' + \lambda T = 0
\end{aligned}
$$